Reasons for Rank-Dependent Utility Evaluation

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Abstract

Three reasons for why people may evaluate utility in a rank-dependent fashion have been suggested: (a) rank-dependent weighting is a function of perceptual biases and thus not prescriptively defensible; (b) weights are (re)distributed by motivational processes that reflect stable personality characteristics of the decision maker; and (c) weights are (re)distributed as a function of the situation, allowing rank-dependent evaluation to be a rational response to an environment with asymmetric loss functions. By modifying a study by Wakker, Erev, and Weber (1994) we show that all three processes—that is, perceptual biases, individual predispositions in weighting, as well as rational adaptation to an asymmetric loss function—can be involved in rank-dependent weighting.

Key words: comonotonic independence, expected utility, independence, preference reversals, rank-dependent utility

A variety of rank-dependent utility (RDU) models have been proposed in recent years as solutions to different theoretical and empirical problems with expected utility models and their extensions. RDU, which assumes that the weight assigned to outcomes is a function of their rank in the distribution of possible outcomes, is currently probably the most popular alternative to expected utility (EU) as a model of risky decision making, though several rank-dependent models (e.g., Luce & Fishburn, 1991; Tversky & Kahneman, 1992) also have a sign-dependent component to which they owe a good portion of their descriptive success. Quiggin (1982) developed RDU to eliminate the violations of stochastic dominance implied by choice models like subjective expected utility (Edwards, 1962) and prospect theory (Kahneman & Tversky, 1979), which transform the probabilities of each outcome in a nonlinear and nonadditive fashion. Quiggin showed that the transformation of cumulative (or decumulative) probabilities, which forces the probability weights to add to one, provided the additional constraint that eliminated violations of stochastic dominance, an insight that was subsequently incorporated into a cumulative version of prospect theory (Tversky & Kahneman, 1992). Luce and Narens (1985) investigated how general the representation of utility could be and still retain the desirable property of interval scalability and found that an RDU representation was the answer, an idea that was generalized to more than two outcomes by Luce (1988). Birnbaum (1973, 1974) introduced rank-dependent weighting as a special case of configural weighting in information integration tasks and subsequently advocated it to explain buying and selling prices for uncertain prospects (Birnbaum & Stegner, 1979) as well as risky choice (e.g., Birnbaum, Coffee, Mellers, & Weiss, 1992; Birnbaum & Sutton, 1992). Lopes (1984, 1987, 1990) independently arrived at an RDU account of risky choice behavior to explain empirical regularities observed by her in a series of studies. Empirical support for RDU has also been provided by Cho, Luce, and von Winterfeldt (1994) and Chung, von Winterfeldt and Luce (1994).

1. Reasons for rank-dependent weighting

RDU weighting can be described either as the utility representation resulting from a set of axioms, most of which have a rational character (Luce, 1990), or simply as an equation that captures how the evaluation of one outcome depends on the value of other possible outcomes. In this section, we describe three simple processes that may lead people to evaluate utility in a rank-dependent fashion. All three operate by drawing attention to some outcomes more than others, as a function of the outcomes' ranking in the distribution of possible outcomes, but they do so for different reasons.

The first type of explanation was suggested by Tversky and Kahneman (e.g., 1986), who hypothesized that the differential weighting of outcomes is a function of *perceptual* biases. The *S*-shaped rank-dependent weighting function of cumulative prospect theory (Tversky & Kahneman, 1992) places disproportionate weight on both the highest and the lowest outcome since they are assumed to be more perceptually salient by virtue of being positioned at the end points of the distribution. A perceptual bias explanation of RDU evaluation implies that rank-dependent weighting misrepresents true utilities, that is, that RDU may be descriptively superior but prescriptively inferior to EU.

The other two explanations assume that attention is drawn not by perceptual processes, but by *motivational* processes. Based on empirical evidence from a series of experiments using lotteries with many possible outcomes, Lopes (1984, 1987, 1990) concluded that individuals differ in the relative emphasis they put on outcomes at the low (security) end of the distribution or at the high (potential) end of the distribution, and that this tendency is a stable, dispositional, *individual-difference* characteristic. Security-minded individuals are assumed to overweight outcomes at the low end of the distribution, whereas potential-minded individuals do the opposite. The maximin and maximax models of decision making under ignorance (e.g., Arrow & Hurwicz, 1972) are extreme cases of such behavior, where all the weight goes to the worst or best outcome, respectively.

A *motivational* explanation for rank-dependent weighting in risky choice of the *situational variety* has been suggested by Birnbaum et al. (1992), who showed that rank-dependent weighting can be derived by minimizing an asymmetric loss function for over- versus underestimates of the value of risky choice alternatives. Loss functions describe the consequences that over- and underestimates of the value of an uncertain quantity will bring to the forecaster or decision maker, after the true value is revealed or after the actual outcome is obtained. According to this situationally determined loss-function interpretation of rank-dependent weighting, changes in the situation that affect loss functions should cause a corresponding change in the pattern of rank-dependent weighting. This hypothesis correctly predicted rank-order differences in the pricing of risky options in Birnbaum and Stegner (1979) and Birnbaum et al. as a function of whether participants had been asked to take a buyer's point of view, a seller's point of view, or a neutral point of view (to state a "fair price" for the buyer to pay the seller). Buyers were assumed to overweight low outcomes, because the consequences of overestimating the true value (e.g., having a better chance at getting the option, but paying too much for it) are more severe than the consequences of underestimates (e.g., having a smaller chance at getting the option, but having saved money). Sellers were assumed to overweight high outcomes, because the consequences of underestimating the true value (e.g., having a better chance at selling the option, but getting too little for it) are more severe than the consequences of overestimates (e.g., having a smaller chance at selling the option, but making money on the deal). Economic theory assumes that buyers and sellers operate from different levels of wealth on the utility function, and the point-of-view manipulation should thus affect the utilities (see Harless, 1989). However, observed differences between buyers' and sellers' prices are usually too large to be explained by a wealth effect via utilities (Knetsch & Sinden, 1984).

Asymmetric loss-function explanations of RDU weighting have prescriptive implications. If RDU evaluation is a response to internal or external constraints rather than the result of a perceptual bias, then it is not necessarily a misrepresentation of true utility. If RDU occurs as the result of sensitivity to asymmetric losses, then the debate about its status as a descriptive versus prescriptive theory reduces to the question of whether the constraints imposed by asymmetric loss functions can or should be ignored by the decision maker.

2. Rationale of experiment

The experiment presented in section 3 was designed to provide tests of the three processes described in the previous section as possible causes of RDU evaluation. The logic of the experiment is to manipulate different variables, of which some can be assumed to result in differences in perceptual salience and others in situational differences in point of view with resulting differences in loss functions. If attention is attracted by some outcomes simply because of their surface characteristics (e.g., being extreme or occurring for sure), then variables that manipulate the consequences of making different types of mistakes (i.e., vary the loss function for over- vs. underjudging the final outcome of a lottery) should not affect people's judgments or choices. Instead, responses should be affected by information displays that redirect attention without changing the consequences of people's answers. If, on the other hand, attention is directed simply by subsequent consequences (i.e., asymmetric loss functions), then perceptual manipulations of attention should show no effect. As with many alternative explanations, it seems reasonable that both perceptual *and* motivational situational factors affect the distribution of attention. In addition to perceptual salience effects and effects of situationally induced asymmetry of consequences of over- vs. underestimates, respondents may also have stable trans-situational dispositions to pessimistically overweight outcomes of low rank or to optimistically overweight outcomes of high rank.

3. Experiment

We extended and modified the experimental paradigm suggested by Wakker, Erev, and Weber (1994) who showed that the difference between independence versus comonotonic independence is the crucial axiomatic demarcation between EU and RDU. The *independence* condition requires that preference between two alternatives not be influenced by outcomes common to both alternatives. As a result, changes in the value of a common outcome should not affect preference. A weaker version of the independence condition allows changes in the value of a common outcome to have an effect, in so much as the change in value affects the rank order of the other outcomes of the two alternatives. Pairs of gambles like the ones shown in table 1, where the outcomes of the S and the R gamble depend on events that induce the same rank order in the outcomes, are called comonotonic. *Comonotonic independence* requires that, for comonotonic gambles that have a common outcome, preference should not be affected by changes in the value of the

Table 1. Stimulus sets 1, 2, and 3

Set 1	Events		
	1	2	3
Safer gamble S	CO	\$6.00	\$7.00
Riskier gamble R	CO	\$4.50	\$10.50
	$p(E_1) = .55$	$p(E_2) = .25$	$p(E_3) = .20$
CO = \$0.50, \$3.50, \$6.50, \$11.50			
$EV_R - EV_S = +\$0.32$			
Set 2	**Events**		
	1	2	3
Safer gamble S	CO	\$2.50	\$6.00
Riskier gamble R	CO	\$1.50	\$7.50
	$p(E_1) = .40$	$p(E_2) = .40$	$p(E_3) = .20$
CO = \$0.50, \$3.00, \$5.50, \$8.00			
$EV_R - EV_S = -\$0.10$			
Set 3	**Events**		
	1	2	3
Safer gamble S	CO	\$4.00	\$4.00
Riskier gamble R	CO	\$2.00	\$5.00
	$p(E_1) = .50$	$p(E_2) = .10$	$p(E_3) = .40$
CO = \$2.00, \$4.00, \$6.00, \$8.00			
$EV_R - EV_S = +\$0.20$			

Adapted from Wakker, Erev, and Weber (1994).

common outcome if the change in common outcome does not alter the ranks of the other outcomes. If a change in the value of the common outcome modifies the ranks of the other outcomes, preference may change. Depending on the hypothesized nature of the rank-dependent weighting function (e.g., optimistic overweighting of high outcomes, pessimistic overweighting of low outcomes, or overweighting of both high and low outcomes because of their extremity at the expense of intermediate outcomes), different RDU theories will predict different patterns of noncomonotonic independence violations.

3.1 Stimuli

Wakker, Erev, and Weber (1994) proposed the following experimental design as the most basic and robust test between EU and RDU. The design tests for the difference between independence and comonotonic independence and, by presenting gambles in a "states-of-nature" format, is not affected by violations of other assumptions, in particular the reduction principle and transitivity. People are presented with gamble pairs of the kind shown in table 1, consisting of a "safer" gamble *S* and a "riskier" gamble *R* with similar expected values (EV) but different variance. Both gambles have three possible outcomes, which are contingent on the same events that occur with well-specified probabilities (e.g., selecting a ball of a given color from an urn with a known number of balls of different colors). One outcome has the same value in each pair (CO, for common outcome). A set of gamble pairs consists of four pairs that are identical except for the value of the common outcome. The CO is either the lowest, middle, or highest outcome for both gambles, making the two gambles in each pair comonotonic. For set 1 shown in table 1, for example, the CO was \$0.50 for the first pair (the lowest outcome for both gambles), \$3.50 for the second pair (still the lowest outcome), \$6.50 for the third pair (the middle outcome), and \$11.50 for the fourth pair (now the highest outcome). For all pairs *S* and *R*, it was true that $R_{lo} < S_{lo} \leq S_{hi} < R_{hi}$; that is, there were no dominating alternatives.

If people evaluate the choice alternatives according to EU, the independence condition will hold, and preference for either the *S* or the *R* gamble in the four pairs of each set should be the same, that is, should not be affected by the value of the common outcome. If people evaluate the choice alternatives according to RDU, on the other hand, only comonotonic independence needs to be satisfied. Preference should not be different for the first and second gamble pair of set 1, since the change in the value of the common outcome does not change the rank order of outcomes (i.e., the CO is the lowest outcome in both pairs). When changes in the value of the common outcome modify the rank order of the other outcomes, however, (i.e., from the second to the third and from the third to the fourth choice pair), preference is allowed to change. Somebody with a pessimistic rank-dependent weighting scheme that gives disproportionate weight to the lowest possible outcome, for example, may prefer the *R* gamble in the first and in the second choice pair, where the lowest outcome is common to both gambles and the *R* gamble offers a greater upside; the same person may switch to the *S* gamble for the third choice pair, since now the *S* gamble offers the greater lowest outcome.

The three sets of four lottery pairs used in our study and shown in table 1 were randomly selected from the six sets used by Wakker, Erev, and Weber (1994). For one of them (set 1), we expanded the range of possible outcomes from (\$0.50, \$9.50) in the original study to (\$0.50, \$11.50), to see whether the absence of statistically significant evidence for RDU evaluation in the original study was due to an insufficient difference in the range of outcomes between the *R* and *S* gambles. The other two sets were exactly the same as in the previous study, except for the additional between-subject motivational and perceptual manipulations described later. Wakker et al. presented the gamble pairs in several display formats, but found that display format had no effect on respondents' preference. Thus we only used their graphical display condition, since it was best suited for manipulating the perceptual salience of outcomes.

In addition to the original choice task where gambles were presented in a pairwise fashion, we also presented each gamble individually and asked respondents for a pricing judgment, as described later. This task was added to see whether the original Wakker, Erev, and Weber (1994) design, while basic and robust, may possibly be susceptible to the following problem. Since the common outcomes in each pair are so transparently common, people may simply cancel them. Such cancellation was observed by Wu (1994). If people cancel the COs in the choice pairs of the Wakker et al. design, the four pairs within each set become identical, and preference should not differ, except for random error in responding. Replicating the original choice task, but also asking respondents to evaluate the same gambles one at a time in a pricing task, allows us to distinguish between the following two interpretations of the fact that Wakker et al. found no significant difference in the proportion of violations of monotonic and nonmonotonic independence. According to the first interpretation, the result is evidence that people do not use RDU weighting when evaluating risky prospects. According to the second interpretation, respondents may very well use RDU weighting when evaluating risky prospects, but cancel transparently common outcomes in risky option pairs. If so, RDU weighting will show in their responses to the pricing task where they evaluate prospects one at a time and common outcome cancellation is not applicable.

3.2 Respondents

Ninety-two members of the University of Chicago community were recruited through campus posters to participate in a gambling study.

3.3 Design and procedure

Participants were told that the purpose of the study was to find out what kind of lotteries people find attractive, and that they would be asked to make choices between pairs of lotteries and to assign prices to individual lotteries. Remuneration for participation depended on their choices and prices as described later. The computer-administered experiment was self-paced.

Lotteries were represented in the graphical format shown in figure 1. Each of the three outcomes of a lottery was represented by a solidly colored box with a height proportionate to the probability of the outcome and a width proportionate to the size of the payoff. The magnitude of payoffs was also printed numerically to the right of the box. The outcomes were shown in descending order of magnitude from top to bottom. Participants were instructed that the outcome of a lottery would be determined by the color of the ball drawn at random from an urn with 100 balls, which had the indicated distribution of balls of the three colors. For the lottery pairs, they were told that the same random draw would determine the color of the ball for both lotteries. Lotteries differed in the number of balls of each color (i.e., in the probabilities of different outcomes) and in the magnitude of the payoffs. All lotteries had only positive payoffs, which ranged from \$0.50 to \$11.50. Expected values ranged from \$2.30 to \$9.54.

In the first part of the study, participants were shown 36 pairs of lotteries, one pair at a time, and had to indicate which lottery they would prefer to play. The two lotteries were shown side by side as lotteries A and B, with the question "Which lottery would you prefer to play?" underneath. Respondents typed in the letter of the preferred lottery. Indifference could not be expressed. The twelve target lottery pairs listed in table 1 were shown twice, together with 12 filler pairs. Following Wakker, Erev, and Weber (1994), the filler pairs had a similar structure as the target pairs but clearly distinct EVs, in order to

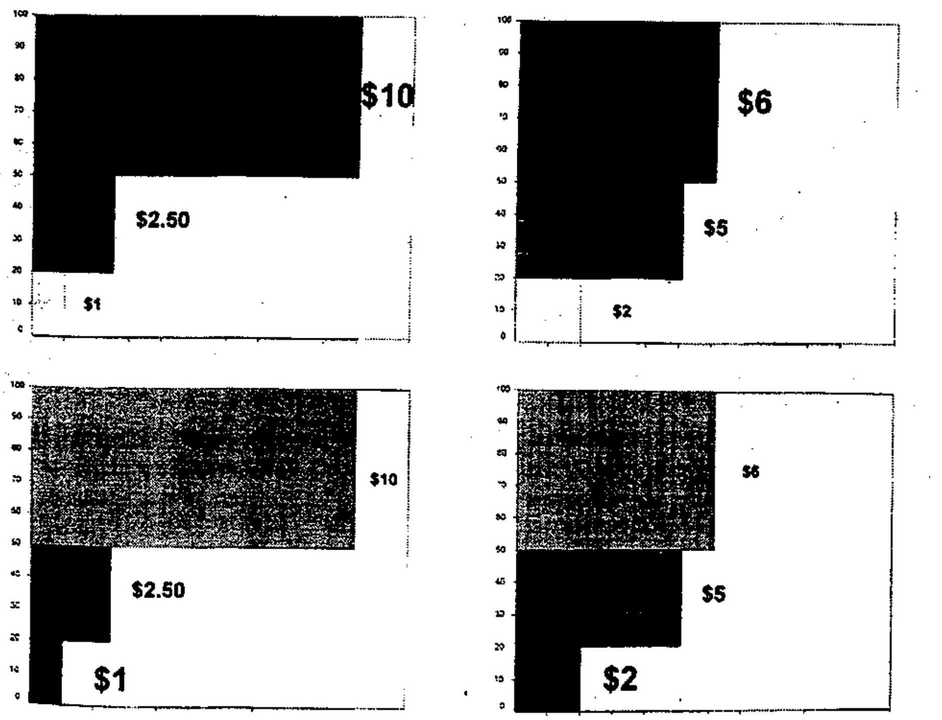

Figure 1. Sample lottery pairs under the two perceptual salience manipulations.

discourage participants from merely choosing at random between the lotteries. The order in which the lottery pairs appeared was random and different for each subject, as was the placement of the lotteries on the right and left side of the screen.

In the second part of the study, participants saw 48 individual lotteries, one at a time, and were asked to express how they valued it by assigning it a price. The 48 lotteries consisted of the S and R lotteries of the 12 target pairs, with half of them repeated as a consistency check, and one lottery from each of the 12 filler pairs.

Perceptual manipulation. For half of the participants, the graphical display of the lotteries for both the pairwise choices and the price judgments emphasized the highest outcome of each lottery by using a much larger font for the display of the numerical value and a brighter color for the corresponding box, as shown in figure 1(a). For the other half of the participants, the lowest outcomes of the lotteries were thus highlighted, as shown in figure 1(b).

Motivational manipulation. Crossed with the between-subject perceptual manipulation was a between-subject motivational manipulation, designed to affect people's weighting scheme as the result of asymmetric loss functions. Half of the respondents were told that they owned the lottery and had a chance to *sell* it. The instructions stated that they should choose an amount of money that would make them indifferent between receiving this amount for sure versus playing out the lottery and receiving the resulting outcome as payment. The other half of respondents were told that they had a chance to *buy* each lottery. They were to choose an amount of money for which they would be indifferent between paying it to be able to play the lottery and to receive the resulting outcome versus not paying the amount and not getting the opportunity to play the lottery. Based on the results of Birnbaum and Stegner (1979) and Birnbaum et al. (1992), we hypothesized that buyers would overweight the lowest outcomes at the expense of higher ranked outcomes when setting their prices for the lotteries, since erring on the high side (overpaying for a lottery) would hurt more than erring on the low side (not getting to play a lottery). For the opposite reason, we hypothesized that sellers would overweight the highest outcomes.

To provide people with some guidance for their pricing judgments, we implemented the Becker, DeGroot, and Marschak (BDM) (1964) payoff mechanism for the seller's condition and an adaptation of the mechanism for the buyer's condition, as described later. This was done mainly to operationalize the buyer and seller roles and as a way of establishing a similar "market" environment for both buyers and sellers. As discussed by Keller, Segal, and Wang (1993), the truth-revealing property of the BDM mechanism depends on the assumption that people are EU maximizers.

Respondents were randomly assigned to one of the four conditions of this 2 $\times$ 2 between-subject experimental design. The entire set of instructions was given to them at the beginning of the session. Consequently respondents knew during the choice task executed first that they would be in the position of either a seller or a buyer of lotteries during the subsequent pricing task. At the conclusion of the experimental session, one of the pairs of lotteries from the choice part of the study was randomly selected for respondents in the seller's condition, and the preferred lottery from the pair was played out. For respondents in the buyer's condition, two such pairs of lotteries were selected, and for each pair the preferred lottery was played out. The money won in these lotteries was payed to the participants. Buyers received the earnings of two lotteries at this stage, because their expected earnings from the second stage, described next, where they either bought a lottery and played it or failed to buy one were approximately zero. Sellers, on the other hand, also had positive expected earnings (either the purchase price or the outcome of the lottery) at the second stage. As a second payoff stage, 1 of the 48 lotteries for which respondents set a selling or buying price in the second part of the study was randomly selected. The computer generated a random number between the highest and lowest outcome of the lottery as its buying or selling price offer. In the seller's condition, if the computer was willing to buy the lottery for more than the respondent's indifference price, the transaction went through; that is, the respondent received the buying price and gave up the right to play the lottery. If the buying price was below the stated indifference price, respondents played out the lottery and received the money they won. In the buyer's condition, if the computer was willing to sell the lottery for less than the respondent's buying price, the respondent paid the computer and received the lottery, which was then played out and the amount won was payed to the respondent. If the computer's selling price was greater than the respondent's buying price, nothing happened. Respondents were informed about the payment scheme at the beginning of the session and knew that they would not receive any other compensation for their participation.

4. Results

4.1 Consistency

Each respondent chose twice between the lotteries of the 12 experimental target pairs, which allowed us to examine the consistency of an individual's choices. On average, people were consistent 73% of the time across repeated choices (i.e., chose the *S* or the *R* gamble both times). This average across respondents was somewhat misleading, however, in light of a bimodal distribution, with some participants responding essentially randomly and most responding more consistently than the mean. Twenty subjects were excluded from further analysis for responding randomly to the choices, that is, for having a repeated-choice consistency between 40% and 60%. (All major results reported later also held when the data from all 92 respondents were analyzed.) Many of the subjects who randomly responded to the choices also gave the same price to all gambles in the pricing task, suggesting that they either did not understand the instructions or were trying to finish the experiment as quickly as possible. For the remaining 72 respondents, the mean proportion of consistent choices was .81.

4.2 Preference

The proportions of *R* preference across respondents and the four experimental conditions are shown in table 2. Preference was either expressed by direct choice or inferred from the difference in prices given to the *R* and the *S* member of a given pair, with the assumption that a higher price implied preference. The literature on preference reversals provides ample evidence that these two ways of expressing relative preference may not result in the same rank order of alternatives (see discussion later). Our question was, more specifically, whether one or both of these preferences would show evidence of RDU evaluation. Preferences were marginally different when expressed by direct choice than when inferred from differences in price ($F(1, 68) = 3.14, p < 10$). Relative preference for the *R* gamble differed significantly by set ($F(2, 136) = 43.44, p < .0001$), with most *R* preferences for set 1, less for set 3, and least for set 2. Choice-based preference was closely related to the difference in EV between the *R* and *S* gamble of each pair. The difference between EV_R and EV_S was +\$0.30 for the pairs of set 1, −\$0.10 for set 2, and +\$0.20 for set 3. The correlation between these EV differences and the proportions of *R* preferences in the choice task shown in table 3 was very strong ($r(11) = .97, p < .0001$). The same correlation was smaller for the proportions of *R*-preferences inferred from the pricing task ($r(11) = .70, p < .05$).

If people decided between the *S* and *R* lotteries of a pair based on an EV or EU evaluation of the two gambles, choice proportions should not be significantly different for the four pairs within a set. If the decision is made, on the other hand, based on a RDU evaluation, choice proportions can be different for pairs within a set, in particular when changes in the value of the common outcome modify the ranks of the other outcomes. For the choice proportions shown in table 2, there was a significant rank-of-common-outcome by task interaction effect ($F(2, 69) = 5.00, p < .01$), such that choice proportions did not significantly differ as a function of the rank of the common outcome in the choice task (.58 for low, .50 for middle, and .57 for high common outcomes), but *did* differ signifi-

Table 2. Proportion of *R* preferences across respondents and experimental conditions

		Task	
Set	Common outcome	Choice	Pricing
1	Lowest	.76	.57
	Lowest	.73	.64
	Middle	.77	.38
	Highest	.86	.53
2	Lowest	.21	.34
	Middle	.26	.30
	Middle	.32	.34
	Highest	.28	.45
3	Lowest	.53	.69
	Middle	.64	.51
	Highest	.61	.57
	Highest	.56	.53

Table 3. Proportion of preferences for *R* gamble across respondents and sets as a function of experimental manipulations

(a) Motivational manipulation

	Preference	
Motivation	Choice-inferred	Price-inferred
Buyer	.54	.43
	|	|[a]
Seller	.55	.56

(b) Perceptual manipulation

	Preference	
Emphasis on	Choice-inferred	Price-inferred
Highest outcome	.58	.53
	|[a]	|[a]
Lowest outcome	.51	.45

[a]The difference in proportions was significantly different from zero at the .05 level.

cantly (.51 for low, .40 for middle, and .56 for high common outcomes) when inferred from pricing. This result is consistent with the cancellation-of-common-outcomes hypothesis, that is, with the assumption that people will cancel transparently common outcomes when encountering them in the choice task, which leaves the four choice pairs of each set looking identical after the cancellation operation. Since people set prices for lotteries by considering them one at a time, cancellation does not apply. The largest effect of rank of common outcome on price-inferred preference occurred for set 1, which was the set in which we had increased the range of outcomes for the *R* gamble to increase the power of the design to detect deviations from EU evaluation.

Table 3 shows the effect of the two experimental manipulations on preference. Both the motivational manipulation (buyer vs. seller perspective) ($F(1, 68) = 6.51, p < .05$) and the perceptual salience manipulation ($F(1, 68) = 4.84, p < .05$) affected preference. Relative preference for the *R* gamble was greater for sellers than for buyers, a difference that was significant only for price-inferred preference for which the different motivational conditions had been operationalized. Consistent with our prediction that sellers would overweight the highest outcome of lotteries, they were more likely to find the *R* gamble with its wider range of outcomes (and thus with a greater upside as well as downside) preferable over the *S* gamble (i.e., assigning it a higher price) than buyers. Relative preference for the *R* gamble was also greater when the highest rather than the lowest outcomes of the lotteries were visually emphasized, consistent with the hypothesis that unequal attention to different outcomes as the result of perceptual aspects of the choice alternatives may drive rank-dependent weighting.

4.3 Prices

The buying and selling prices provided by respondents for the *R* and *S* gambles of each pair were strongly affected by the motivational manipulation ($F(1, 68) = 42.67$, $p < .0001$), with sellers setting higher prices than buyers. In addition to this main effect, motivation also interacted with the type of gamble for which a price was set ($F(23, 1564) = 6.95$, $p < .0001$). Being in the seller's rather than buyer's motivational condition raised prices disproportionately more for *R* gambles (where the difference was $1.47) than for *S* gambles (where the difference was $1.10). This difference in differences had the effect that for buyers, *S* gambles were valued more than *R* gambles ($3.71 > $3.52), but that for sellers, *S* gambles were valued less than *R* gambles ($4.81 < $4.99). The difference in the prices assigned to *R* and *S* gambles when either the highest or lowest outcome was visually emphasized was not sufficient to result in a significant effect of perceptual salience on prices ($F < 1$).

4.4 Comonotonic versus noncomonotonic independence violations

Consistent with the fact that differences in preference for the *R* gamble in the choice task as a function of the rank of the common outcome were not large enough to be significant, there was only a marginally significant difference in the rate of violations of comonotonic versus noncomonotonic independence for the choice task ($F(1, 68) = 3.45, p < .10$). (For this and all other analyses of proportions, the data first underwent an arcsine transformation to satisfy ANOVA assumptions.) As shown in table 4, violations of comonotonic independence occurred 25% of the time and violations of noncomonotic independence 29% of the time in the choice task. For preferences inferred from prices, there was a significant difference in the rate of comonotonic versus noncomonotonic violations of independence ($F(1, 68) = 4.62, p < .05$). Both violation rates were somewhat higher for preference inferred indirectly from prices (with opportunity for random error in both price judgments) than for preferences expressed directly by choice (with only a single source of random error). But even with a higher rate of random error reflected by violations of comonotonic independence (28%), these violations occurred significantly less frequently than violations of noncomonotonic independence (35%), which reflect random error plus changes in preference due to shifts in the weights given to outcomes.

For both expressions of preference (choices and prices), there was an interaction between type-of-violation and motivational condition ($F(1, 68) = 6.83$ and 12.83, respectively, $p < .01$). Violations of comonotonic independence were significantly less frequent than violations of noncomonotic independence for buyers (24% vs. 31% when expressed by choice, and 24% vs 37% when expressed by relative prices) but did not differ significantly for sellers (26% vs. 27% when expressed by choice, and 32% vs 32% when expressed by relative prices). This suggests that the condition asking respondents to evaluate lotteries from a buyer's perspective was more effective in inducing rank-dependent weighting than the seller's condition. Members of the general public may have more experience with a buyer's perspective when deciding on prices, so that buyer's

Table 4. Proportion of violations of comonotonic and noncomonotonic independence as a function of experimental manipulations

(a) Task

Task	Independence: Comonotonic		Independence: Noncomonotonic
Choices	.25		.29
Pricing	.28	—[a]—	.35

(b) Motivational manipulation

Motivation	Independence in choices: Comonotonic		Independence in choices: Noncomonotonic	Independence in prices: Comonotonic		Independence in prices: Noncomonotonic
Buyers	.24	—[a]—	.31	.24	—[a]—	.37
Sellers	.26		.27	.32		.32

(c) Perceptual manipulation

Emphasis	Independence in choices: Comonotonic		Independence in choices: Noncomonotonic	Independence in prices: Comonotonic		Independence in prices: Noncomonotonic
Lowest outcome	.22	—[a]—	.30	.26	—[a]—	.35
Highest outcome	.28		.29	.30	—[a]—	.35

[a]The difference in proportions was significantly different from zero at the .05 level.

instructions would reinforce existing weighting schemes, whereas seller's instructions would counteract and possibly conflict with those schemes.

The manipulation of perceptual salience had the following effect on violation rates: Random error was greater when the highest outcome rather than the lowest outcomes of the gambles was visually emphasized ($F(1, 68) = 4.05$, $p < .05$). That is, violations of monotonic independence increased from 22% to 28% for choices and from 26% to 30% for prices, suggesting that people may have a natural tendency to focus attention more on the lowest outcomes. The highest outcome perceptual salience condition would interfere with this tendency, leading to greater random error. The results of the individual-subject fits of different special cases of RDU models, shown in table 5(a) and discussed in the next section, are consistent with this interpretation, showing that the largest group of respondents across experimental conditions used a pessimistic weighting scheme that overweighted low outcomes. Table 4 also shows that for prices, both perceptual salience conditions were successful in inducing rank-dependent weighting, that is, resulted in a significantly greater rate of violations of noncomonotonic than comonotonic independence (35% vs. 26% for the low-outcome emphasis, and 35% vs. 30% for the high-outcome emphasis). For the choices, the difference was only significant for the low-outcome emphasis (30% vs. 22%).

When violations of independence were examined at the individual subject level, we identified three types of response patterns. The first pattern, exhibited by 17% of respondents, showed a small but equal proportion of comonotonic and noncomonotonic inde-

Table 5. (a) Frequency of respondents whose choices and price judgments were jointly best fit by indicated utility models and (b) probability of models for each experimental condition

(a)	Models				
Condition	EU	CPT	RDU_{opt}	RDU_{pes}	
Buyers, high-outcome emphasis	3	4	7	9	23
Buyers, low-outcome emphasis	2	1	2	10	15
Sellers, high-outcome emphasis	4	2	5	6	17
Sellers, low-outcome emphasis	3	3	6	5	17
	12	10	20	30	72
	17%	14%	27%	42%	

(b)	Models			
Condition	EU	CPT	RDU_{opt}	RDU_{pes}
Buyers, high-outcome emphasis	.13	.17	.30	.39
Buyers, low-outcome emphasis	.13	.07	.13	.67
Sellers, high-outcome emphasis	.24	.12	.30	.35
Sellers, low-outcome emphasis	.18	.17	.35	.29

pendence violations in both their choices and their price judgments. These respondents were true EU maximizers. The second pattern, exhibited by 39% of respondents, showed an equal proportion of both independence violations in their choice judgments, but had a larger proportion of noncomonotonic than comonotonic independence violations for their price judgments. This pattern is consistent with RDU maximizers who cancelled the CO in the choice pairs. The third pattern, exhibited by the remaining 44% of respondents, showed a larger proportion of noncomonotonic than comonotonic independence violations for both their choices and their price judgments. This pattern is consistent with RDU maximizers who did *not* cancel the CO in the choice pairs.

4.5 Best-fitting special cases of RDU

Table 5(a) provides a classification of respondents with respect to the utility model that best described both their choices and their price data. The 17% of respondents who showed an equal proportion of violations of comonotonic or noncomonotonic independence in both their choices and their price-inferred preferences were classified as EU maximizers. For each of the remaining respondents, the observed violations of noncomonotonic independence were classified as to whether or not they constituted violations according to special cases of rank-dependent outcome weighting. In particular, we considered the following weighting schemes: pessimistic overweighting of low outcomes, optimistic overweighting of high outcomes, and cumulative prospect theory (CPT) overweighting of extreme outcomes. A full description of the preference changes allowed and disallowed under each special RDU model for the three choice sets can be found in figures 3.1, 3.3, and 3.6 of Wakker, Erev, and Weber (1994). For set 1, for example, pessimistic overweighting of the lotteries' lowest outcomes allows for a change in preference from R to S (but not S to R) as the common outcome is changed from $3.50 (lowest) to $6.50 (middle), whereas the opposite is true for optimistic overweighting of the lotteries' highest outcomes. Formal arguments for these predictions are provided in Wakker et al. Conceptually, the predictions are based on particular redistributions of weights (according to the weighting schemes of the RDU special cases) to events that have different outcomes for the R and S lotteries. If a change in the rank of the common outcome results in greater weight given to an event for which R has a better outcome than S, then a shift in preference from S to R is allowable, but not the opposite. If, on the other hand, a change in the rank of the common outcome gives more weight to an event for which S has a better outcome than R, preference can shift from R to S, but not the other way.

The resulting proportion of violations of noncomonotonic independence *not* allowed by each of the three RDU weighting schemes allowed us to classify the preferences of each respondents as being most consistent with one of the three rank-dependent utility models listed in table 5(a). The RDU model that fit the choices and prices of each respondent best was the model that resulted in the lowest rate of nonallowed noncomonotonic independence violations. Across respondents, this rate was no longer significantly different from their rate of comonotonic independence violations.

As shown in table 5(a), the largest group of respondents (42% across experimental conditions) showed preferences most consistent with a pessimistic rank-dependent weighting scheme. The next largest group (27%) showed optimistic rank-dependent weighting. Only 14% of respondents had preferences most consistent with the extremity weighting described by cumulative prospect theory, while 17% had preferences most consistent with EU maximization.

Using a loglinear analysis of the frequencies shown in table 5(a), we examined whether the number of respondents whose preferences were best fit by each of the four utility models differed as a function of the motivational and perceptual salience condition they found themselves in during the experiment. Consistent with other results described earlier, the situational motivation of being either in a buyer's or seller's position had a stronger effect on which model provided the best fit (chi-square(3) = 3.03) than the perceptual salience manipulation (chi-square(3) = 0.76), which had an effect only in the interaction with motivation (chi-square(3) = 2.84). The direction of these effects is apparent in table 5(b), which provides the probabilities of observing the four models in each experimental condition. The pessimistic RDU model is most likely (.67) when both the motivational and perceptual manipulation encourage emphasis of the lowest outcome. Under that condition, the optimistic RDU model is also least likely to be observed (.13). The extremity overweighting of both lowest and highest outcome of the CPT model is most likely to occur when the motivational manipulation draws attention to one extreme outcome and the perceptual manipulations draws attention to the other outcome. None of these effects is, however, as strong as the intercept effect, that is, a main effect of model preference regardless of experimental condition (chi-square(3) = 12.03, $p < .01$), the only effect to reach statistical significance.

4.6 Consistency in selection between choices and pricing—preference reversals

Even though repeated-choice consistency was quite high (i.e., 81% of times nonrandomly responding participants chose the same alternative when being presented with the same choice pair, and the average correlation between prices for the 12 gambles presented twice was .86), consistency of preference *between* elicitation modes was lower. Consistency between the preferred alternative indicated by people's choice and the preferred alternative inferred from their pricing of the two choice alternatives was only 46% across respondents and choice pairs. This means that 54% of the time, preferences reversed between being assessed by choice and by pricing. There was a significant effect of perceptual salience ($F(1, 860) = 10.44, p < .005$) as well as an interaction between perceptual salience and motivation ($F(1, 860) = 4.09, p < .05$). Preference reversals were more frequent when the lowest outcomes rather than highest outcomes of the gambles were emphasized (with reversal rates of 60% vs. 49%), but this effect occurred mostly for sellers (61% vs. 42%) and less for buyers (58% vs. 54%).

These results provide further evidence for the hypothesis that many respondents edited choice pairs by cancelling the CO. After cancellation, the value of the CO for the pairs within a set does not affect preference. The value of the CO entered however into pricing judgments and affected price-based preference, in ways that are consistent with respondents evaluating the utility of the alternatives in a rank-dependent fashion. The difference between choice-based and price-based preference, and thus the rate of preference reversals, was largest for those cases where all three determinants of rank-dependent utility evaluation pushed the weighting function into the same direction, resulting in strong rank-dependent preference that showed in price-based preference but less in choice-based preference due to the cancellation of the CO. As discussed in the previous section, respondents were more likely to be predisposed to pessimistically overweight low outcomes than to use any other kind of weighting scheme. When this personal predisposition toward pessimism was reinforced by making the lowest outcome perceptually more salient than other outcomes, preference reversals were at their peak (60%). Preference reversals were much reduced (42%) when the natural tendency toward pessimism was counteracted both perceptually (visually emphasizing the highest outcome) and by situational motivation (asking people to take a seller's perspective).

5. Discussion

5.1 Explanations of the Wakker, Erev, and Weber (1994) null result

The experimental design of the study reported in this paper was developed to detect differences in the rate of comonotonic versus noncomonotonic independence violations and thus to discriminate between EU and RDU evaluation. While Wakker, Erev, and Weber (1994) failed to find significant differences in the overall rate of comonotonic versus noncomonotonic independence violations, their null results were open to explanations other than the conclusion that RDU may not provide a better descriptive model than

EU. The current modification of their study was designed to test whether the cancellation of common outcomes in choice pairs may be partially responsible for their null results.

Wu (1994) provided some empirical support for the assumption that people edit choice pairs by cancelling common outcomes when the commonality is obvious. The commonality of outcomes in the choice pairs of the experimental design suggested by Wakker, Erev, and Weber (1994) is about as obvious as it possibly can be. If people cancel the common outcomes, the four pairs within each set become identical, and choice will not depend on the rank of the common outcome regardless of whether respondents use EU or RDU to evaluate the alternatives. In our study, 39% of respondents showed evidence of cancelling common outcomes in the choice task, leading to no difference in the violation rates of comonotonic versus noncomonotonic independence despite the fact that they evaluated gambles in a rank-dependent fashion in their price judgments. Contrary to the implications of Wakker et al.'s null results, only 17% of the respondents in our study made both choice and price judgments consistent with EU. It is, of course, possible (though perhaps not very plausible) that people maximize EU in their choices but RDU when making price judgments. This interpretation should be tested with additional studies that find ways to discourage CO cancellation in choice pairs. To minimize people's inclination to cancel common outcomes, the experimental gamble pairs could be interspersed with a large number of filler pairs that do not contain any common outcomes, for example. This was done by Birnbaum and McIntosh (1995), who found evidence of rank-dependent weighting in choices. At the very least, our results suggest that the experimental paradigm for testing between EU and RDU suggested by Wakker et al. needs to be applied and interpreted with considerable care.

5.2 RDU and choice-price preference reversals

The reversals between choice-based and price-inferred preference observed in this study were, at least partly, due to the fact that a significant portion of respondents seemed to cancel the COs of the lottery pairs in the choice task but incorporated them into their pricing judgments. If people were EU maximizers, such cancellation could not lead to a reversal in the rank order of preference between the two elicitation modes. When alternatives are evaluated in a rank-dependent fashion, on the other hand, CO cancellation can have this effect.

Even for choice pairs that share no common outcomes, RDU evaluation may contribute to commonly observed choice-price preference reversals (e.g., Lichtenstein and Slovic, 1971) in the following way. Asking people to provide selling prices for lotteries induces a situational motivation to overweight high outcomes, leading to greater preference for lotteries with the best highest outcome (i.e., the $-bets of Lichtenstein and Slovic, 1971). The choice task, on the other hand, is more neutral in its situational motivation for differential weighting of outcomes as a function of their rank; thus, people's predisposition or perceptual biases will determine the weighting scheme. Given that the common subject population for preference reversal studies (i.e., students or members of university communities) seem to be predisposed to use a pessimistic rank-dependent weighting scheme (see table 5(a)), low outcomes will receive greater weight in choice, leading to preference for lotteries that guarantee a greater minimum outcome (e.g., the P-bets of Lichtenstein and Slovic, 1971; also see Lopes, 1987).

5.3 Reasons for RDU evaluation

Our study found significantly fewer violations of comonotonic than noncomonotonic independence for the price-inferred preferences and a marginally significant trend in that direction for the choices, suggesting that, under some circumstances, RDU can provide a better description of utility evaluation than EU. In addition, our study was designed to test whether different mechanisms that have been directly or indirectly suggested in the literature were sufficient in influencing the direction and extent of rank-dependent weighting.

Perceptual biases. Our manipulation of the perceptual salience of the lowest versus highest outcome of lotteries was effective in the sense that it altered choices and price-inferred preference. Even though perceptual bias effects were weaker than motivational effects, these results cannot be generalized beyond the manipulations of our study. In addition, our experimental conditions that visually highlighted either the highest or lowest outcomes of gambles may not have been as strong as the natural bias, hypothesized by Tversky and Kahneman (1986), of paying greater attention to both high- and low-ranked rather than intermediate outcomes, because of their extremity. Our perceptual and motivational manipulations that differentially emphasized high- and low-ranked outcomes may, in fact, have interfered with the weighting postulated by CPT that gives greater weight to *both* extreme outcomes. Overall, only 14% of our respondents showed the CPT weighting pattern. The importance of our results therefore lies in the demonstration that perceptual salience *can* affect the distribution of rank-dependent weights.

Situational asymmetric loss functions. Putting respondents into the position of having to set either buying or selling prices for lotteries in order to earn their take-home pay strongly affected price judgments. More than a simple main effect, differences in the rank order of prices between the two groups (i.e., buyers valuing S gambles more than R gambles, sellers doing the opposite) could be explained by differences in the rank-dependent weighting functions, which were consistent with plausible differences in post-decisional regret on the part of buyers and sellers.

Personal disposition and individual differences. In addition to rank-dependent weighting induced by the perceptual salience of outcomes and by being placed in a buyer or seller role, we also found strong evidence of individual differences in utility weighting that transcended these situational constraints. Across conditions, the most common weighting scheme was the pessimistic overweighting of low-ranked outcomes, followed by the optimistic overweighting of high-ranked outcomes. These results are not necessarily inconsistent with the weighting function proposed by Tversky and Kahneman (1992). As mentioned earlier, our perceptual salience manipulation that directed attention at *either* the lowest *or* the highest possible outcome of the choice alternatives may have disrupted people's natural tendency to overweight both extreme outcomes. Furthermore, the S-shaped weighting function of cumulative prospect theory is not symmetric around .5, redistributing more weight to lower ranked outcomes than to higher ranked outcomes, consistent with our finding of a tendency toward pessimism. For example, the weight of three equally likely outcomes, ordered from lowest to highest, is predicted by cumulative prospect theory to be .49, .18, and .33, a shift of weight from the middle to the lowest outcome.

5.4 Significance and implications

What do different interpretations of rank-dependent weighting contribute to our understanding of judgment and decision making under risk and uncertainty? If RDU evaluation occurs as the result of perceptual biases, the model may be descriptively accurate, but is prescriptively indefensable (Edwards, 1992; Howard, 1992). If, on the other hand, RDU evaluation is a response to internal or external constraints, then it is not necessarily a misrepresentation of true utility.

In addition to being subject to internal (i.e., psychological) asymmetric costs, people frequently face external asymmetric consequences; for example, they may be held responsible for outcomes that fall short of the predicted outcome in business decisions, while outcomes that exceed the expected outcome are taken for granted and barely noticed. Hogarth and Kunreuther (1992) provide some reasons why EU may be insufficient to capture, for example, the real and asymmetric constraints (e.g., tax codes) under which actuaries operate when pricing insurance policies for risks with ambiguous losses (i.e., losses with uncertain probabilities or uncertain amounts).

As a result, a decision analyst dealing with a client who is currently evaluating options in a rank-dependent fashion has to ask him or herself *why* the evaluation is rank-dependent in this case. If rank-dependent weighting is the result of an attentional bias where some outcomes receive more weight than they should, based on the likelihood of their occurrence, because of the way information about them is being presented, the analyst needs to design ways to equalize attention to all possible outcomes. If rank-dependent weighting persists, after such perceptual or attentional intervention, the post-decisional consequences of different outcomes for the decision maker ought to be explored.

If RDU occurs as the result of sensitivity to asymmetric losses, then the debate about its status as a descriptive versus prescriptive theory reduces to the question of whether the constraints imposed by asymmetric loss functions can or should be ignored by the decision maker. Decision aiding will involve helping people to ask themselves what internal constraints they are imposing onto themselves, and what external constraints other people in their decision environment (e.g., spouse, peers, superior) are imposing on them. Only when they are aware of all consequences of over-versus underestimating the uncertain quantity, can people decide which of these consequences *should* affect their decision.

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